Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY

MANIPAL

(A constituent unit of MAHE, Manipal)

FIRST SEMESTER M.TECH (SOFTWARE ENGINEERING) DEGREE END SEMESTER EXAMINATION-NOVEMBER 2018

SUBJECT: MATHEMATICAL LOGIC (ICT 5122)

(REVISED CREDIT SYSTEM)

TIME: 3 HOURS | 22/11/2018 | MAX. MARKS: 50

Instructions to candidates

- Answer **ALL** questions. All questions carry equal marks.
- Missing data if any, may be suitably assumed.

1A. Consider the argument "*If it has snowed, it will be poor driving. If it is poor driving, I will be late unless I start early. Indeed, it has snowed. Therefore, I must start early to avoid being late*". Use the following atoms:

s : it has snowed

p : it is poor driving

l : I will be late, and

e : I start early

to write the given argument as the sequent in propositional logic. Prove the sequent using natural deduction rules. [5]

1B. Consider the sequent $p \vee q, p \rightarrow r \vdash r$. Determine a DAG which is not satisfied iff this sequent is valid. Tag the DAG's root node with '1:T,' apply the forcing laws to it, and extract a witness to the DAG's satisfiability. [3]

1C. Prove the following theorem of propositional logic.

$$((p \rightarrow q) \rightarrow q) \rightarrow ((q \rightarrow p) \rightarrow p)$$

[2]

2A. Prove the validity of the sequent

$$\exists x \exists y (H(x,y) \vee H(y,x)), \neg \exists x H(x,x) \vdash \exists x \exists y \neg (x = y).$$

[5]

2B. Use the predicate specifications

$B(x,y)$: x beats y

$F(x)$: x is a football team

$Q(x,y)$: x is quarterback of y

$L(x,y)$: x loses to y

c : Wildcats, and

j : Jayhawks.

to translate the following into predicate logic

i) Every football team has a quarterback.

ii) If the Jayhawks beat the Wildcats, then the Jayhawks do not lose to every football team.

iii) The Wildcats beat some team, which beat the Jayhawks.

[3]

2C. Let ϕ be the sentence $\forall x \forall y \exists z (R(x, y) \rightarrow R(y, z))$, where R is a predicate system of two arguments

i) Let $A \triangleq \{a, b, c, d\}$, and $R^{\mathcal{M}} \triangleq \{(b, c), (b, b), (b, a)\}$. Do we have $\mathcal{M} \vDash \phi$? Justify your answer, whatever it is.

ii) Let $A' \triangleq \{a, b, c\}$ and $R^{\mathcal{M}'} \triangleq \{(b, c), (a, b), (c, b)\}$. Do we have $\mathcal{M}' \vDash \phi$? Justify your answer, whatever it is.

[2]

3A. Write the pseudo-code for a function SAT, which takes a CTL formula as input and returns the set of states satisfying the formula. [5]

3B. Consider the system of Figure Q.3B. For each of the formulas ϕ:

(a) Ga (b) aUb (c) $aUX(a \wedge \neg b)$

i) Find a path from the initial state q_3 which satisfies ϕ.

ii) Determine whether $\mathcal{M}, q_3 \vDash \phi$.

[3]

3C. Find operators to replace the ?, to make the following equivalences:

i) $AG(\phi \wedge \psi) \equiv AG\phi ? AG\psi$

ii) $EF\neg\phi \equiv \neg ?? \phi$

[2]

4A. Using the natural deduction rules for $KT45^n$, prove the validity of

i) $K_i(p \wedge q) \leftrightarrow K_i p \wedge K_i q$

ii) $C(p \wedge q) \leftrightarrow Cp \wedge Cq$

[5]

4B. Write formulas for the following:

i) Agent 1 knows that p.

ii) Agent 1 knows that p or q.

iii) Agent 1 knows p or agent 1 knows q.

iv) Agent 1 knows whether p.

v) Agent 1 knows whether agent 2 knows whether p.

vi) Some people know p but don't know q.

[3]

4C. Write the natural deduction proof for the following sequent over the basic modal logic K.

$$\vdash_K \vdash \Box(p \to q) \land \Box(q \to r) \to \Box(p \to r)$$

[2]

5A. With reference to NuSMV, explain the following

i) Inclusion operator
ii) Case expression
iii) If-Then-Else expression
iv) Basic next expression
v) Count operator

[5]

5B. Describe different types of variable declaration in a finite state machine in the NuSMV language. [3]

5C. Write syntax of LTL formulas recognized by NuSMV. [2]

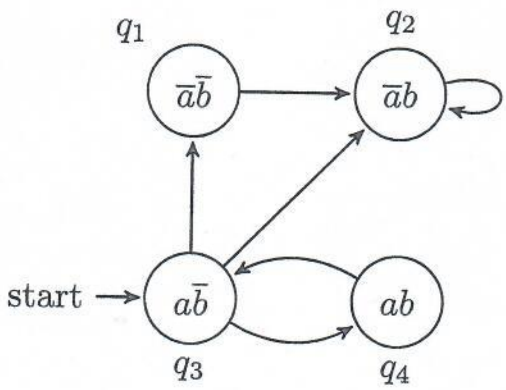

Figure: Q.3B